

Formal Semantics: Introduction

We now parlay our understanding of the formal language, and its connection with English, into a **semantic** test of validity – beginning with some introductory observations on what a “semantic” test involves.

Semantics forms a somewhat hybrid discipline, focusing as it does on two different aspects of a language: (i) the **meaning** of sentences in that language, and (ii) the **truth and falsehood behavior** of those sentences – which situations make a given sentence true, and which make it false. But those two seemingly unrelated topics are in fact remarkably closely connected.

On the one hand, knowing the meaning of a sentence is typically a requirement for knowing whether that sentence is true or false in a given situation. Looking out the window, you can establish the truth or falsehood of the sentence “It’s raining,” but not of the sentence “It’s zhaqti” – because you understand the meaning of first sentence, but not of the second.

But that order can be reversed. If your travels take you to a remote tribe having no previous contact with outsiders, and speaking an unknown language, building the first English-Tribalese dictionary will require you to figure out the meanings of sentence (and their parts) in this new language. For this you would naturally consider which situations make a given sentence true, and which don’t. So if speakers utter the sounds “Gavagai!” when faced with running or roasted rabbits, but not when confronted with dogs or diamonds, we guess that this little sentence means something like “It’s a rabbit” or “There’s a rabbit”. Here the truth-and-falsehood behavior of the sentence yields its meaning.¹

Applying this double-barreled discipline to issues of validity, we have two routes open to us: develop a theory of meaning for the language of logical form, or develop a theory of truth and falsehood. But the definitions of “valid argument” and “validity counterexample” themselves make clear it’s the second route which interests us.

¹ The example is from the logician W.V. Quine, who used it for rather different purposes.

Valid Argument: an argument where, every time the premises are all **true**, the conclusion is also **true**

Validity Counterexample: a possible situation where the premises are all **true**, but the conclusion is still **false**.

Since the concepts of truth and falsehood are central to both definitions, we will grasp semantics by this horn, and develop a **general theory of truth and falsehood** for our formal language. But given the afore-mentioned connection between the two sides of semantics, this choice sacrifices nothing. For one especially happy by-product of this theory of truth will be a formal theory of sameness-of-meaning.

By aiming for a **general** semantic theory, we mean: a theory which provides a truth and falsehood profile for every sentence in the formal language. The secret to ensuring this universal coverage lies in sentence construction.

Recall that every formal sentence is constructed in strict compliance with the four construction rules of the language.

1. Sentence letters are formal sentences.
2. If \bullet is a formal sentence, then $\sim \bullet$ is a formal sentence.
3. If \bullet and \blacktriangle are formal sentences, then $(\bullet \wedge \blacktriangle)$ is a formal sentence.
4. If \bullet and \blacktriangle are formal sentences, then $(\bullet \vee \blacktriangle)$ is a formal sentence.

By building a theory of truth and falsehood for each of these four types of sentences, we can be sure that no formal sentence falls outside the scope of the semantic theory.